Automatic continuity for groups from GGT (j.w.w. Daniel Keppeler and Olga Varghese)

Philip Möller

7.10.2021

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$,

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?

Partial answer: If G is a group from GGT and L is locally compact Hausdorff, this seems to be possible in many cases.

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?
Partial answer: If G is a group from GGT and L is locally compact Hausdorff, this seems to be possible in many cases.
Convention: G carries the discrete topology.

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?

Partial answer: If G is a group from GGT and L is locally compact Hausdorff, this seems to be possible in many cases.

Convention: G carries the discrete topology.

Goal for today:

- Give an overview of the results.

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?

Partial answer: If G is a group from GGT and L is locally compact Hausdorff, this seems to be possible in many cases.

Convention: G carries the discrete topology.

Goal for today:

- Give an overview of the results.
- Explain why GGT groups show up here.

Motivation and Introduction

Question: Given two *topological* groups L, G and a homomorphism $\varphi\colon L \to G$, can we find conditions on the **groups** ensuring that φ is continuous?

Partial answer: If G is a group from GGT and L is locally compact Hausdorff, this seems to be possible in many cases.

Convention: G carries the discrete topology.

Goal for today:

- Give an overview of the results.
- Explain why GGT groups show up here.
- Connection to Buildings.

Historical Results

Theorem (Dudley 1961)

Any homomorphism from a locally compact Hausdorff group into a free (abelian) group is continuous.

Historical Results

Theorem (Dudley 1961)

Any homomorphism from a locally compact Hausdorff group into a free (abelian) group is continuous.

Theorem (Morris-Nickolas, 1976)

Any abstract group homomorphism from a locally compact Hausdorff group into a free product of groups is continuous or the image of the homomorphism is contained in a conjugate of one of the factors.

Historical Results

Theorem (Dudley 1961)

Any homomorphism from a locally compact Hausdorff group into a free (abelian) group is continuous.

Theorem (Morris-Nickolas, 1976)

Any abstract group homomorphism from a locally compact Hausdorff group into a free product of groups is continuous or the image of the homomorphism is contained in a conjugate of one of the factors.

Theorem (Dudley/Conner-Corson 2019/Kramer-Varghese 2019)

Any group homomorphism $\varphi\colon L \to A_\Gamma$ from a locally compact Hausdorff group into a right-angled Artin group is continuous.

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Theorem (Corson-Varghese 2019)

A group G is lcH-slender if and only if it is torsion free and does not contain $\mathbb{Q}$ or the group of p-adic integers $\mathbb{Z}_p$.

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Theorem (Corson-Varghese 2019)

A group G is lcH-slender if and only if it is torsion free and does not contain $\mathbb{Q}$ or the group of p-adic integers $\mathbb{Z}_p$.

Is this the "best" we can do?

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Theorem (Corson-Varghese 2019)

A group G is lcH-slender if and only if it is torsion free and does not contain $\mathbb{Q}$ or the group of p-adic integers $\mathbb{Z}_p$.

Is this the "best" we can do?
Yes (it is an "iff"-Theorem)...

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Theorem (Corson-Varghese 2019)

A group G is lcH-slender if and only if it is torsion free and does not contain $\mathbb{Q}$ or the group of p-adic integers $\mathbb{Z}_p$.

Is this the "best" we can do?
Yes (it is an "iff"-Theorem)...
...and no, e.g. the Theorem of Morris and Nickolas also gives information about groups containing torsion or $\mathbb{Q}$ or $\mathbb{Z}_p$.

Historical Results

Definition (Conner-Corson 2019)

A group G is called *lcH-slender* if every homomorphism $\varphi\colon L \to G$ from a locally compact Hausdorff group L to G is continuous.

Theorem (Corson-Varghese 2019)

A group G is lcH-slender if and only if it is torsion free and does not contain $\mathbb{Q}$ or the group of p-adic integers $\mathbb{Z}_p$.

Is this the "best" we can do?
Yes (it is an "iff"-Theorem)...
...and no, e.g. the Theorem of Morris and Nickolas also gives information about groups containing torsion or $\mathbb{Q}$ or $\mathbb{Z}_p$.

General Theme

If G is rich in structure then $\varphi\colon L \to G$ is continuous or the image is "small".

More History

Theorem (M.-Varghese 2020)

Let $\Phi\colon L \to \mathrm{Isom}(X)$ *denote a group action from a locally compact group* L *on a CAT*(0) *space of finite flat rank. If*

More History

Theorem (M.-Varghese 2020)

Let $\Phi\colon L \to \mathrm{Isom}(X)$ denote a group action from a locally compact group L on a CAT(0) space of finite flat rank. If fixed point sets of single elements and of closed subgroup behave "nicely", then the pattern holds.

More History

Theorem (M.-Varghese 2020)

Let $\Phi\colon L \to \mathrm{Isom}(X)$ *denote a group action from a locally compact group* L *on a CAT*(0) *space of finite flat rank. If fixed point sets of single elements and of closed subgroup behave "nicely", then the pattern holds. In this case* Φ *is either continuous or there is a non-empty proper invariant subset* $\mathrm{Fix}(\Phi(H))$ *for* $H \leq L$ *a closed subgroup.*

More History

Theorem (M.-Varghese 2020)

Let $\Phi \colon L \to \mathrm{Isom}(X)$ denote a group action from a locally compact group L on a CAT(0) space of finite flat rank. If fixed point sets of single elements and of closed subgroup behave "nicely", then the pattern holds. In this case Φ is either continuous or there is a non-empty proper invariant subset $\mathrm{Fix}(\Phi(H))$ for $H \leq L$ a closed subgroup.

Question: Does this pattern continue for groups acting nicely on a more general metric space, say a Helly group?

More History

Theorem (M.-Varghese 2020)

Let $\Phi \colon L \to \mathrm{Isom}(X)$ denote a group action from a locally compact group L on a CAT(0) space of finite flat rank. If fixed point sets of single elements and of closed subgroup behave "nicely", then the pattern holds. In this case Φ is either continuous or there is a non-empty proper invariant subset $\mathrm{Fix}(\Phi(H))$ for $H \leq L$ a closed subgroup.

Question: Does this pattern continue for groups acting nicely on a more general metric space, say a Helly group? Or more generally a metrically injective group?

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

(i) *G does not include $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ for any prime p as a subgroup,*

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

(i) *G does not include $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ for any prime p as a subgroup,*

(ii) *G does not include the Prüfer p-group $\mathbb{Z}(p^\infty)$ for any prime p as a subgroup,*

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

(i) *G does not include $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ for any prime p as a subgroup,*

(ii) *G does not include the Prüfer p-group $\mathbb{Z}(p^\infty)$ for any prime p as a subgroup,*

(iii) *torsion subgroups in G are artinian,*

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

(i) *G does not include $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ for any prime p as a subgroup,*

(ii) *G does not include the Prüfer p-group $\mathbb{Z}(p^\infty)$ for any prime p as a subgroup,*

(iii) *torsion subgroups in G are artinian,*

then any group homomorphism $\varphi\colon L \to G$ from a locally compact group L to G is continuous, or there exists a normal open subgroup $N \subseteq L$ such that $\varphi(N)$ is a non-trivial torsion group.

Results for today

Theorem (Keppeler-M.-Varghese, 2021)

Let G be a discrete group. If

(i) *G does not include $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ for any prime p as a subgroup,*

(ii) *G does not include the Prüfer p-group $\mathbb{Z}(p^\infty)$ for any prime p as a subgroup,*

(iii) *torsion subgroups in G are artinian,*

then any group homomorphism $\varphi\colon L \to G$ from a locally compact group L to G is continuous, or there exists a normal open subgroup $N \subseteq L$ such that $\varphi(N)$ is a non-trivial torsion group.
If additionally

(iv) *G does not have non-trivial torsion normal subgroups,*

then every surjective $\varphi\colon L \to G$ is continuous.

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*
2. *a cocompactly cubulated* $\mathrm{CAT}(0)$ *group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*
2. *a cocompactly cubulated $\mathrm{CAT}(0)$ group,*
3. *a $\mathrm{CAT}(0)$ group whose torsion subgroups are artinian, e.g. a Coxeter group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*
2. *a cocompactly cubulated $\mathrm{CAT}(0)$ group,*
3. *a $\mathrm{CAT}(0)$ group whose torsion subgroups are artinian, e.g. a Coxeter group,*
4. *a Gromov-hyperbolic group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*
2. *a cocompactly cubulated* $\mathrm{CAT}(0)$ *group,*
3. *a* $\mathrm{CAT}(0)$ *group whose torsion subgroups are artinian, e.g. a Coxeter group,*
4. *a Gromov-hyperbolic group,*
5. *a metrically injective group whose torsion subgroups are artinian, e.g. a Helly group whose torsion subgroups are artinian,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

If G is a subgroup of

1. *a virtually lcH-slender group,*
2. *a cocompactly cubulated* $\mathrm{CAT}(0)$ *group,*
3. *a* $\mathrm{CAT}(0)$ *group whose torsion subgroups are artinian, e.g. a Coxeter group,*
4. *a Gromov-hyperbolic group,*
5. *a metrically injective group whose torsion subgroups are artinian, e.g. a Helly group whose torsion subgroups are artinian,*
6. *a finitely generated residually finite group whose torsion subgroups are artinian, e.g. the (outer) automorphism group of a right-angled Artin group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7 *a one-relator group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7 *a one-relator group,*

8 *a finitely generated linear group in characteristic* 0*,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7. *a one-relator group,*
8. *a finitely generated linear group in characteristic* 0*,*
9. *the Higman group,*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7. *a one-relator group,*
8. *a finitely generated linear group in characteristic* 0*,*
9. *the Higman group,*

then any group homomorphism $\varphi\colon L \to G$ *from a locally compact group* L *is continuous or there exists a normal open subgroup* $N \subseteq L$ *such that* $\varphi(N)$ *is a torsion group.*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7. *a one-relator group,*
8. *a finitely generated linear group in characteristic* 0*,*
9. *the Higman group,*

then any group homomorphism $\varphi\colon L \to G$ *from a locally compact group* L *is continuous or there exists a normal open subgroup* $N \subseteq L$ *such that* $\varphi(N)$ *is a torsion group.*
If G *does not have non-trivial torsion normal subgroups, then every surjective* $\varphi\colon L \to G$ *is continuous.*

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7. *a one-relator group,*
8. *a finitely generated linear group in characteristic* 0,
9. *the Higman group,*

then any group homomorphism $\varphi\colon L \to G$ *from a locally compact group* L *is continuous or there exists a normal open subgroup* $N \subseteq L$ *such that* $\varphi(N)$ *is a torsion group.*
If G *does not have non-trivial torsion normal subgroups, then every surjective* $\varphi\colon L \to G$ *is continuous.*

Proofidea for the Theorem.

Combine Iwasawa's Structure theorem (connected case) and van Dantzig's Theorem (t.d. case).

Results for today

Corollary (Keppeler-M.-Varghese 2021)

7. *a one-relator group,*
8. *a finitely generated linear group in characteristic* 0*,*
9. *the Higman group,*

then any group homomorphism $\varphi\colon L \to G$ *from a locally compact group* L *is continuous or there exists a normal open subgroup* $N \subseteq L$ *such that* $\varphi(N)$ *is a torsion group.*
If G *does not have non-trivial torsion normal subgroups, then every surjective* $\varphi\colon L \to G$ *is continuous.*

Proofidea for the Theorem.

Combine Iwasawa's Structure theorem (connected case) and van Dantzig's Theorem (t.d. case). Study compact groups, abelian divisible groups and torsion groups and their images.

Why GGT groups?

Given a group G and an action $\Phi\colon G \to \mathrm{Isom}(X)$ we can often use the properties of the action to show that certain subgroups can't exist:

Why GGT groups?

Given a group G and an action $\Phi\colon G \to \mathrm{Isom}(X)$ we can often use the properties of the action to show that certain subgroups can't exist:

Definition (Haettel 2021)

A group G is called *metrically injective* if it acts geometrically on an injective metric space, that is a metric space, which is an injective object in the category of metric spaces and 1-Lipschitz maps.

Why GGT groups?

Given a group G and an action $\Phi\colon G \to \mathrm{Isom}(X)$ we can often use the properties of the action to show that certain subgroups can't exist:

Definition (Haettel 2021)

A group G is called *metrically injective* if it acts geometrically on an injective metric space, that is a metric space, which is an injective object in the category of metric spaces and 1-Lipschitz maps.

Example (Injective metric spaces)

The real line,

Why GGT groups?

Given a group G and an action $\Phi \colon G \to \mathrm{Isom}(X)$ we can often use the properties of the action to show that certain subgroups can't exist:

Definition (Haettel 2021)

A group G is called *metrically injective* if it acts geometrically on an injective metric space, that is a metric space, which is an injective object in the category of metric spaces and 1-Lipschitz maps.

Example (Injective metric spaces)

The real line, $\mathbb{R}$-trees,

Why GGT groups?

Given a group G and an action $\Phi \colon G \to \mathrm{Isom}(X)$ we can often use the properties of the action to show that certain subgroups can't exist:

Definition (Haettel 2021)

A group G is called *metrically injective* if it acts geometrically on an injective metric space, that is a metric space, which is an injective object in the category of metric spaces and 1-Lipschitz maps.

Example (Injective metric spaces)

The real line, $\mathbb{R}$-trees, finite dimensional CAT(0) cube complexes with the l^∞ metric,...

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups,

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$,

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$, uniform lattices in Euclidean buildings of type $\tilde{C}_n$,

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$, uniform lattices in Euclidean buildings of type $\tilde{C}_n$, Artin groups of type FC,...

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$, uniform lattices in Euclidean buildings of type $\tilde{C}_n$, Artin groups of type FC,...

Lemma

Let G denote a metrically injective group and $H \leq G$ a subgroup. If H is almost divisible, then H is a torsion group.

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$, uniform lattices in Euclidean buildings of type $\tilde{C}_n$, Artin groups of type FC,...

Lemma

Let G denote a metrically injective group and $H \leq G$ a subgroup. If H is almost divisible, then H is a torsion group. In particular a metrically injective group does not contain $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ as a subgroup.

Why GGT groups?

Example (Metrically injective groups)

Gromov-hyperbolic groups, uniform lattices in $\mathrm{GL}_n(\mathbb{R})$, uniform lattices in Euclidean buildings of type $\tilde{C}_n$, Artin groups of type FC,...

Lemma

Let G denote a metrically injective group and $H \leq G$ a subgroup. If H is almost divisible, then H is a torsion group. In particular a metrically injective group does not contain $\mathbb{Q}$ or the p-adic integers $\mathbb{Z}_p$ as a subgroup. Additionally, a metrically injective group does not contain the Prüfer p-group $\mathbb{Z}(p^\infty)$ as a subgroup.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element. There exists a barycenter map on X which behaves nicely with translation lengths of isometries.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element. There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that $|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.
There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that
$|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.
$\Phi(g)$ is either hyperbolic or elliptic.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.
There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that
$|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.
$\Phi(g)$ is either hyperbolic or elliptic. If it is elliptic, it is a torsion element.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.
There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that
$|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.
$\Phi(g)$ is either hyperbolic or elliptic. If it is elliptic, it is a torsion element. But it can't be hyperbolic because
$\inf\{|\Phi(l)| : \Phi(l) \text{ is a hyperbolic isometry}\} > 0$.

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.
There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that
$|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.
$\Phi(g)$ is either hyperbolic or elliptic. If it is elliptic, it is a torsion element. But it can't be hyperbolic because
$\inf\{|\Phi(l)| : \Phi(l) \text{ is a hyperbolic isometry}\} > 0$.
For the Prüfer-p group show that a metrically injective group only has finitely many conjugacy classes of finite subgroups. $\square$

Why GGT groups?

Proofidea.

Let G be a group acting geometrically on a metrically injective space X and let $g \in G$ denote an almost divisible element.
There exists a barycenter map on X which behaves nicely with translation lengths of isometries. Use that to show that
$|\Phi(g^n)| \geq n|\Phi(g)|$ for all $g \in G$.
$\Phi(g)$ is either hyperbolic or elliptic. If it is elliptic, it is a torsion element. But it can't be hyperbolic because
$\inf\{|\Phi(l)| : \Phi(l) \text{ is a hyperbolic isometry}\} > 0$.
For the Prüfer-p group show that a metrically injective group only has finitely many conjugacy classes of finite subgroups. $\square$

Lemma

A residually finite group does not contain $\mathbb{Q}$ or the Prüfer-p subgroup $\mathbb{Z}(p^\infty)$.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

Proofidea.

For a graph product G_Γ there exists a right-angled building X_Γ on which G_Γ acts isometrically.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

Proofidea.

For a graph product G_Γ there exists a right-angled building X_Γ on which G_Γ acts isometrically. For the "poison subgroups" we can apply our tools from before to reduce to stabilizers of vertices.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

Proofidea.

For a graph product G_Γ there exists a right-angled building X_Γ on which G_Γ acts isometrically. For the "poison subgroups" we can apply our tools from before to reduce to stabilizers of vertices. The artinian subgroup case can also be reduced to vertex stabilizers with a similar argument.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

Proofidea.

For a graph product G_Γ there exists a right-angled building X_Γ on which G_Γ acts isometrically. For the "poison subgroups" we can apply our tools from before to reduce to stabilizers of vertices. The artinian subgroup case can also be reduced to vertex stabilizers with a similar argument. Stabilizers of vertices are induced by complete subgraphs and are thus direct products.

New Examples and Buildings

Proposition

Let Γ be a finite graph and G_Γ denote a graph product of groups. If all the vertex groups satisfy properties (i) - (iii) of the Theorem (no poison subgroup and torsion subgroups are artinian), then so does G_Γ.

Proofidea.

For a graph product G_Γ there exists a right-angled building X_Γ on which G_Γ acts isometrically. For the "poison subgroups" we can apply our tools from before to reduce to stabilizers of vertices. The artinian subgroup case can also be reduced to vertex stabilizers with a similar argument. Stabilizers of vertices are induced by complete subgraphs and are thus direct products. Checking the desired result holds for direct products is an algebra exercise. □

Thank you

Thank you for your attention